Progress in Resummation: Simplicity in All Orders QCD Dynamics

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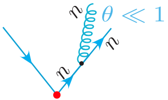
Berkeley and LBNL

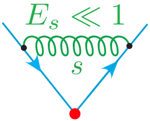
Limits of Gauge Theories

- Gauge theories simplify sufficiently in kinematic limits that they can often be understood to all orders in α_s:

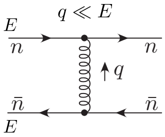
Collinear



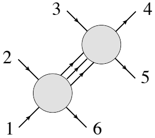
Soft



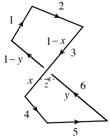
Regge



Factorization



Self Crossing



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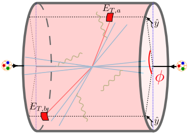
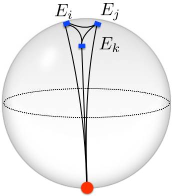
- Many of these correspond to limits of physical interest at the LHC!
  Jet substructure, event shapes, threshold, q_T, BFKL....
- Remarkable continued progress in better understanding this simplicity and in phenomenological applications at colliders.

Outline

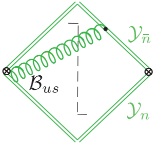
- Precision Hadronic Event Shapes:
  From Drell-Yan to Dijets at NNNLL



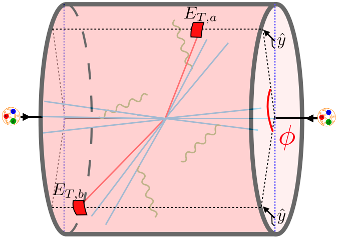
- Towards Simplicity in Jet Substructure:
  Higher Loops and Higher Points



- Extending Resummation Beyond Leading Power



Precision Hadronic Event Shapes: From Drell-Yan to Dijets

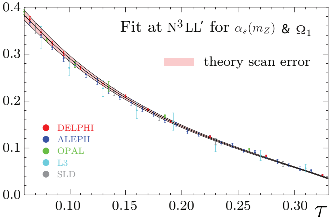


[Gao, Li, IM, Zhu]

Hadronic Event Shapes

- One of the most basic objects of study in QCD are event shapes.
- Well understood in e^+e^-, with results to N^{3}LL +NNLO.

$\frac{\tau}{\sigma}\frac{d\sigma}{d\tau}$ Thrust at N^{3}LL +NNLO



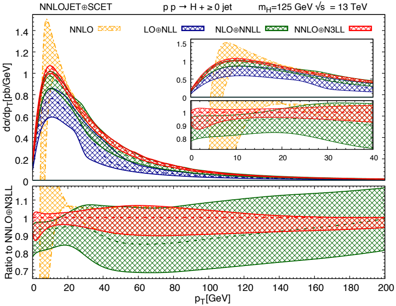
[Abbate, Fickinger, Hoang, Mateu, Stewart]

- Hadron colliders offer a much richer environment:
  - Non-trivial color flows.
  - Factorization violation.
- Progress with colored final states beyond NLL difficult due to the very complicated soft dynamics of multiple colored directions.

A Success Story: q_T

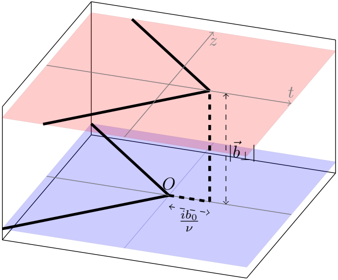
- In the last several years, q_T for color singlet production was computed to N^{3}LL +NNLO.

Higgs q_T Distribution



[Chen, Gehrmann, Glover, Huss, Li, Neill, Schulze, Stewart, Zhu]

q_T Soft Function



[Li, Neill, Zhu]

- Can we extend this success from q_T to dijet event shapes?
- Key is in choosing a "nice" dijet event shape!

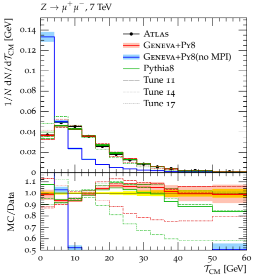
Choosing the Right Observable

- While many event shape observables have been proposed at hadron colliders, these typically suffer from two features that prevent their use as precision observables:

  - Large underlying event sensitivity (factorization violation)

  - Complicated observable definitions for multiple soft emission (e.g. $|E_T|$, N-jettiness, ...)

Beam Thrust

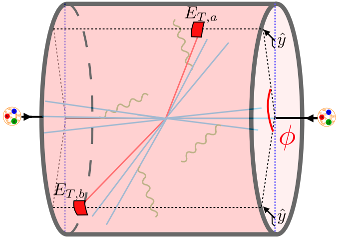


- Would like to overcome this to have precision event shapes at the LHC.

Simplifying Dijet Event Shapes

- Consider the Transverse Energy-Energy Correlator (TEEC)

$$\frac{d\sigma}{d\cos\phi} = \sum_{a,b}\int d\sigma_{pp\to a+b+X}\frac{2E_{T,a}E_{T,b}}{|\sum_i E_{T,i}|^2}\delta(\cos\phi_{ab}-\cos\phi)$$

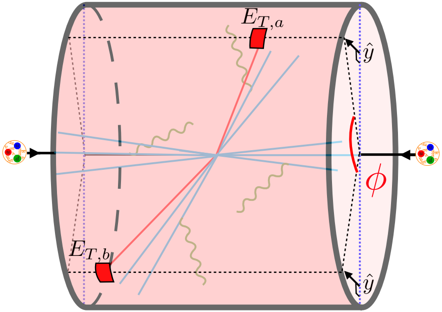


[Basham, Brown, Ellis, Love]

- In the $\tau \equiv \sin^2((\pi-\phi)/2) \to 0$ limit, it is a dijet observable.
- Will show that the TEEC exhibits a remarkable perturbative simplicity, and is quite insensitive to UE.

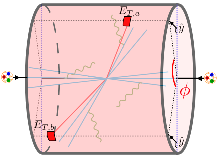
Kinematics

- In the $\tau \to 0$ limit, τ is related to the momentum perpendicular to the plane in which the Born dijets lie.
- Particles out of the plane generated by soft emissions recoiling the plane, or collinear splittings.



- The TEEC is the natural generalization of q_T to dijets: recoiling vector $\to$ recoiling plane.

Factorizaton Formula



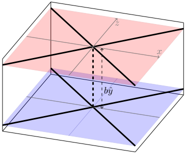
- Factorization for the TEEC in the back-to-back limit:

$$\frac{d\sigma^{(0)}}{d\tau} = \frac{1}{16\pi s^2(1+\delta_{f_3 f_4})\sqrt{\tau}} \sum_{\text{channels}} \frac{1}{N_{\text{init}}} \int \frac{dy_3 dy_4 p_T dp_T^2}{\xi_1 \xi_2} \int_{-\infty}^{\infty} \frac{db}{2\pi} e^{-2ib\sqrt{\tau} p_T}$$

$$\mathrm{tr}\left[\mathbf{H}^{f_1 f_2 \to f_3 f_4}(p_T, y^*, \mu)\mathbf{S}(b, y^*, \mu, \nu)\right] \cdot B_{f_1/N_1}(b,\, \xi_1,\, \mu,\, \nu)\, B_{f_2/N_2}(b,\, \xi_2,\, \mu,\, \nu) J_{f_3}(b, \mu, \nu)\, J_{f_4}(b, \mu, \nu)$$

- Combines a wealth of interesting functions
  - H: $2 \to 2$ Hard Functions (NNLO) [Anastasiou, Bern, De Freitas, Dixon, Glover, ...]
  - B: TMD PDFs (NNLO) [Catani, Echevarria, Gehrmann, Grazzini, Lubbert, Scimemi, Vladimirov, ..]
  - J: (Moment of) TMD fragmentation function (NNLO) [Echevarria, Scimemi, Vladimirov, ..]
  - S: New TEEC Soft Function (NNLO) [Gao, Li, Moult, Zhu]
- And their anomalous dimensions
  - Soft Anomalous Dimension (3-loops) [Almelid, Gardi, Duhr]
  - Rapidity Anomalous Dimension (3-loops) [Li, Neill, Zhu]
  - Collinear Anomalous Dimensions (3-loops) [Moch, Vermaseren, Vogt, ...]
  - Cusp Anomalous Dimension (4-loops) [Korchemsky, ..., Henn, ..., Moch, Vermaseren, Vogt, ...
- Remarkable example of factorization (in both physics and sheer work)!

Soft Function



- Simplicity lies in the TEEC Soft Function

$$\mathbf{S}(b, y^*) = \langle 0 | T[\boldsymbol{O}_{n_1 n_2 n_3 n_4}(0^\mu)] \overline{T}[\boldsymbol{O}^\dagger_{n_1 n_2 n_3 n_4}(b^\mu)] | 0 \rangle$$

- Expanding perturbatively as $\mathbf{S} = \sum (\alpha_s/4\pi)^n \mathbf{S}^{(n)}$

$$\mathbf{S}^{(1)}(y^*, L_b, L_\nu) = -\sum_{i<j} (\mathbf{T}_i \cdot \mathbf{T}_j)\, S_\perp^{(1)}\left(L_b, L_\nu + \ln\frac{n_i \cdot n_j}{2}\right),$$

$$\mathbf{S}^{(2)}(y^*, L_b, L_\nu) = -\sum_{i<j} (\mathbf{T}_i \cdot \mathbf{T}_j)\, S_\perp^{(2)}\left(L_b, L_\nu + \ln\frac{n_i \cdot n_j}{2}\right) + \frac{1}{2!}\left(\mathbf{S}^{(1)}(y^*, L_b, L_\nu)\right)^2$$

- Remarkably, $S_\perp^{(i)}$ is the i loop Color Singlet q_T soft function!!
- Dipole structure preserved at level of cross section.
- First analytic 2-loop dijet soft function.
- Simplicity arises from the fact that the measurement is perpendicular to the plane of the Wilson lines $\implies$ The uniquely simple dijet soft function.

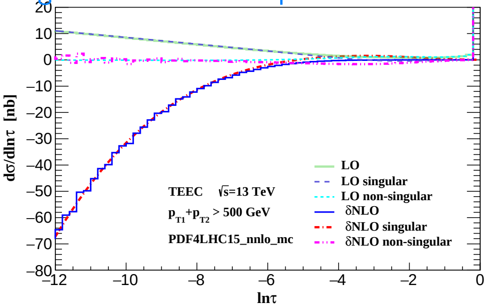
Verification of Singular Behavior

- Factorization formula correctly predicts singular behavior of NLO $pp \to 3$ jet cross section in all partonic channels.

$$\frac{d\sigma^{(0)}}{d\tau} = \frac{1}{16\pi s^2(1+\delta_{f_3 f_4})\sqrt{\tau}} \sum_{\text{channels}} \frac{1}{N_{\text{init}}} \int \frac{dy_3 dy_4 p_T dp_T^2}{\xi_1 \xi_2} \int_{-\infty}^{\infty} \frac{db}{2\pi} e^{-2ib\sqrt{\tau}p_T}$$

$$\mathrm{tr}\left[\mathbf{H}^{f_1 f_2 \to f_3 f_4}(p_T, y^*, \mu)\mathbf{S}(b, y^*, \mu, \nu)\right] \cdot B_{f_1/N_1}(b,\ \xi_1,\ \mu,\ \nu)\, B_{f_2/N_2}(b,\ \xi_2,\ \mu,\ \nu) J_{f_3}(b, \mu, \nu)\, J_{f_4}(b, \mu, \nu)$$

Singular Behavior Compared with NLOJet++

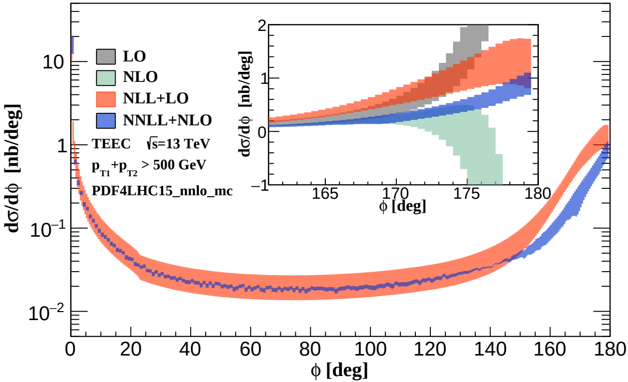


- Analytic control of IR logarithms for NLO $pp \to 3$ jets.

NLO+NNLL Resummation

- First dijet event shape at NNLL:

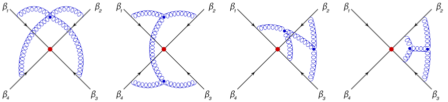
NLO+NNLL TEEC



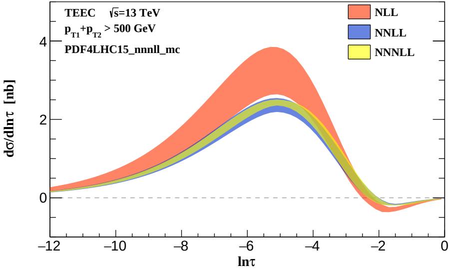
- Resummation has a large effect.
- To improve perturbative behavior, important to go to NNNLL.

NNNLL Resummation

- At NNNLL we encounter for the first time in a physical observable a quadrupole color correlation.
  [Almelid, Duhr, Gardi]



- First (preliminary) results at NNNLL:

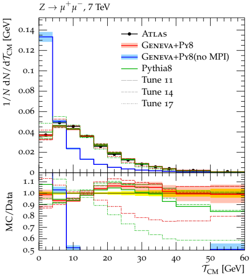


- Must be matched to NNLO $2 \to 3$ amplitudes. See Talks by Abreu, Badger
- e^+e^- level theoretical precision for hadron collider event shapes!

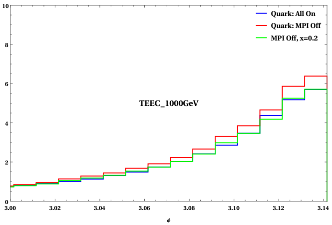
Underlying Event

- Relation to q_T saves the TEEC in another way: Underlying Event does not systematically recoil the plane $\implies$ effects minimal
- Other hadronic event shapes often have $\mathcal{O}(1)$ factorization violation.

Beam Thrust :(



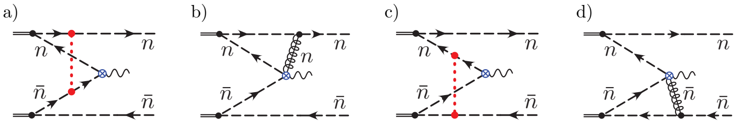
TEEC :)



- UE for TEEC well modeled by adding a uniform energy distribution.
- Can one rigorously prove that this is a power correction (or $1/N_c$ suppressed, or perturbative, or ....)?

Glaubers and Factorization Violation

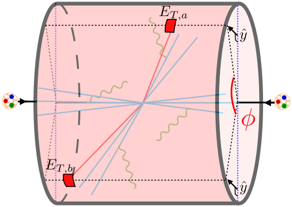
- With colored final states, naive picture of factorization is (generically) violated. [Collins, Catani, Forshaw, ...]
- We have now concretely hit a perturbative accuracy where this must be understood. Factorization formula will explicitly fail.
- Factorization violation can be incorporated using Glauber operators.



[Rothstein, Stewart]

- This is an opportunity: The remarkable simplicity of the TEEC provides a playground to concretely address these issues, and the observed insensitivity to UE provides hope that they can be incorporated.

- The TEEC can be defined for three distinct final states
  - Dijets (factorization violated)
  - $Z/W/\gamma$ + Jet (factorization violated?)
  - Drell-Yan ($Z \to ll$) (factorization proven)

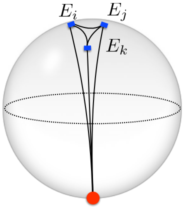


- All three now available at N^{3}LL.

- Allows for precise probe of color flows and factorization violation in a hadron collider environment!

- Hopefully we can learn general lessons about factorization and its violation.

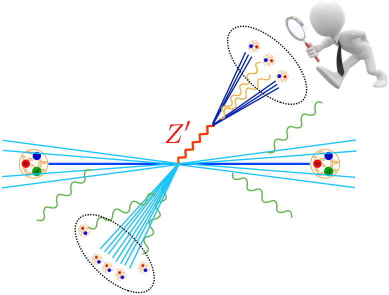
Towards Simplicity in Jet Substructure: Higher Loops and Higher Points

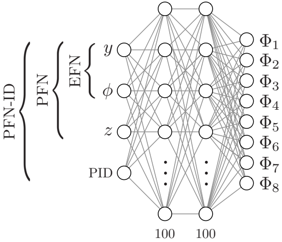


[Dixon, IM, Zhu]
[Chen, Dixon, Luo, IM, Yang, Zhang, Zhu]

Jet Substructure

- Jet Substructure has emerged as a primary way to look for new physics, and probe QCD at the LHC.



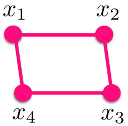


See Jesse Thaler's Talk

- Basic goal of jet substructure is to understand the seemingly complicated correlations in energy flow in a QCD jet.
  - Can we draw inspiration from Conformal Field Theory (CFTs) to find simplicity ?

Back to Basics: Correlation Functions

- The natural observables in a (C)FT are correlation functions. e.g. Four point correlator.



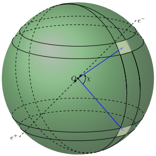
$$\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle = \frac{g(u,v)}{x_{12}^{2\gamma_\phi} x_{34}^{2\gamma_\phi}}$$

$$u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} \qquad v = \frac{x_{23}^2 x_{14}^2}{x_{13}^2 x_{24}^2}$$

Note: $x^\gamma = 1 + \gamma \log x + \frac{1}{2}\gamma^2 \log^2 x + \cdots$

- In a scattering experiment, these operators are placed at infinity, and integrated over time. Primarily measure energy:

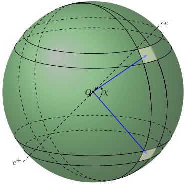
$$\mathcal{E}(\vec{n}) = \int_0^\infty dt \lim_{r\to\infty} r^2 n^i T_{0i}(t, r\vec{n})$$



- The simplest observables are the correlation functions themselves: $\langle \mathcal{O}\mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\cdots\mathcal{E}(\vec{n}_N)\mathcal{O}^\dagger\rangle$

Energy Correlators

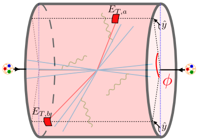
- The simplest observable is the two-point correlator: Energy-Energy Correlator. It can be written in a more familiar way as

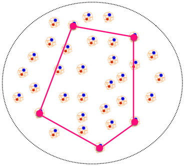


$$\frac{d\sigma}{dz} = \sum_{i,j} \int d\sigma \, \frac{E_i E_j}{Q^2} \delta\left(z - \frac{1-\cos\chi_{ij}}{2}\right)$$

[Basham, Brown, Ellis, Love]

- Naturally generalizes to hadron colliders, and to multi-point correlators.

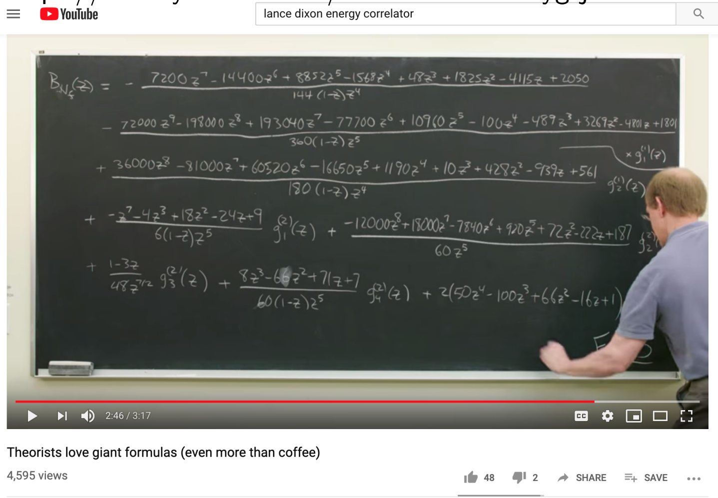




- In the collinear limit, it is a jet substructure observable, with very interesting properties.

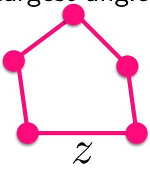
Energy Correlators on Youtube

- The EEC has its own Youtube Video by Lance Dixon: https://www.youtube.com/watch?v=WVC1ygsjZNc



Energy Correlators in a Conformal Field Theory

- Consider an N-point Energy-Energy Correlator in a conformal field theory. Let z denote the largest angle:



- The differential cross section is given by

$$\frac{d\sigma}{dz\ d\text{Shape}} = C_{\text{Shape}}(z=1,\alpha_s)\, z^{\gamma_{N+1}(\alpha_s)-1}$$

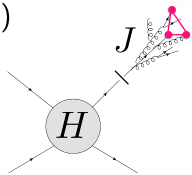
- $C_{\text{Shape}}(z=1,\alpha_s)$ is a (potentially complicated) function describing the shape dependence for a "unit shape".
- $\gamma_{N+1}(\alpha_s)$ is the twist-two spin $N+1$ spacelike anomalous dimension (known at 3 (and for some N, 4) loops in QCD). [Hofman, Maldacena]

[See also: Korchemsky; Kologlu, Kravchuk, Simmons Duffin, Zhiboedov]

Energy Correlators in QCD

- We can derive a factorization formula for the N-point correlator in QCD:

$$\Sigma(z, \ln\frac{Q^2}{\mu^2}, \mu) = \int_0^1 dx\, x^N \vec{J}(\ln\frac{zx^N Q^2}{\mu^2}, \mu) \cdot \vec{H}(x, \frac{Q^2}{\mu^2}, \mu)$$



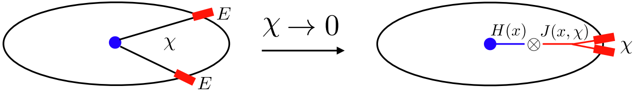
- The jet function satisfies the renormalization group equation:

$$\frac{d\vec{J}(\ln\frac{zQ^2}{\mu^2}, \mu)}{d\ln\mu^2} = \int_0^1 dy\, y^N \vec{J}(\ln\frac{zy^N Q^2}{\mu^2}, \mu) \cdot \widehat{P}_T(y, \mu)$$

- At LL, have exact correspondence with CFT result (up to running coupling): $\vec{J}^T_{LL} = (J_q, J_g) \exp\left(\frac{\gamma(N+1)}{2\beta_0} \ln\frac{\alpha_s(z^{1/N}Q)}{\alpha_s(Q)}\right)$

- In a non-CFT, beyond LL, derivatives $\gamma'(N+1)$, $\gamma''(N+1)$, ... also enter.

The Two Point Correlator



[Dixon, IM, Zhu]

To NNLL in One Slide

- To achieve NNLL (single log counting), one needs all anomalous dimensions at 3 loops, constants at two-loops:
  - Anomalous dimensions can be extracted from [Mitov, Moch, Vermaseren, Vogt, ...]
  - Computed EEC jet functions at NNLO e.g.

$$
\begin{aligned}
j_2^g = {}& n_f^2\left(-\frac{8}{15}\zeta_2+\frac{2344}{1125}\right)+C_F n_f\left(4\zeta_3+\frac{14}{5}\zeta_2-\frac{1528667}{108000}\right)\\
&+C_A n_f\left(\frac{44}{5}\zeta_3-\frac{127}{25}\zeta_2+\frac{68111303}{1620000}\right)+C_A^2\left(76\zeta_4-\frac{1054}{5}\zeta_3-\frac{2159}{75}\zeta_2+\frac{133639871}{810000}\right)
\end{aligned}
$$

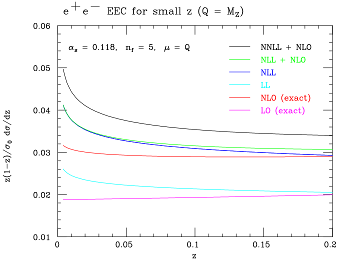
- Control of $\frac{1}{z}$, $\frac{\log z}{z}$, $\frac{\log^2 z}{z}$ at three loops, plus tower of logarithms.
- First substructure observable known at this order.

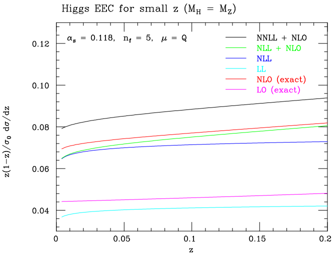
$$
\begin{aligned}
z\Sigma(z) = {}& \frac{\alpha_s}{4\pi}\frac{3C_F}{2}\\
&+\left(\frac{\alpha_s}{4\pi}\right)^2\left[C_AC_F\left(-\frac{50\zeta_2}{3}+4\zeta_3-\frac{107\log(z)}{15}+\frac{35366}{675}\right)+C_Fn_f\left(\frac{53\log(z)}{60}-\frac{4913}{900}\right)\right.\\
&\left.+C_F^2\left(\frac{86\zeta_2}{3}-8\zeta_3+\frac{25\log(z)}{4}-\frac{8263}{216}\right)\right]\\
&+\left(\frac{\alpha_s}{4\pi}\right)^3\left[C_AC_Fn_f\left(\frac{379579\zeta_2}{5400}+\frac{3679\zeta_3}{30}-\frac{118\zeta_4}{3}+\left(-\frac{108\zeta_2}{5}+\frac{16\zeta_3}{3}+\frac{6644267}{54000}\right)\log(z)\right.\right.\\
&\left.-\frac{16259\log^2(z)}{1800}-\frac{1025118113}{2160000}\right)\\
&+C_AC_F^2\left(-\frac{400\zeta_2^2}{3}+\frac{137305\zeta_2}{216}-72\zeta_2\zeta_3+\frac{10604\zeta_3}{15}+\frac{4541\zeta_4}{6}-216\zeta_5\right.\\
&\left.+\left(-\frac{1100\zeta_2}{3}+\frac{262\zeta_3}{3}+\frac{105425}{144}\right)\log(z)-\frac{340}{9}\log^2(z)-\frac{105395741}{51840}\right)\\
&+C_A^2C_F\left(-\frac{906257\zeta_2}{2700}+24\zeta_2\zeta_3-\frac{47483\zeta_3}{90}-\frac{481\zeta_4}{6}+56\zeta_5+\left(\frac{503\zeta_2}{5}-\frac{74\zeta_3}{3}-\frac{2916859}{6750}\right)\log(z)\right.\\
&\left.+\frac{8059\log^2(z)}{300}+\frac{964892417}{540000}\right)\\
&+C_F^2n_f\left(-\frac{15161\zeta_2}{120}-\frac{7994\zeta_3}{45}+\frac{236\zeta_4}{3}+\left(\frac{416\zeta_2}{9}-\frac{32\zeta_3}{3}-\frac{6760183}{64800}\right)\log(z)+\frac{4619\log^2(z)}{720}\right.\\
&\left.+\frac{164829499}{486000}\right)+C_Fn_f^2\left(\frac{6\zeta_2}{5}+\frac{23\log^2(z)}{45}-\frac{8867\log(z)}{1350}+\frac{88031}{4500}\right)\\
&+C_F^3\left(\frac{688\zeta_2^2}{3}-\frac{18805\zeta_2}{216}+48\zeta_2\zeta_3+52\zeta_3-1130\zeta_4+208\zeta_5+\left(\frac{1849\zeta_2}{9}-\frac{172\zeta_3}{3}-\frac{723533}{2592}\right)\log(z)\right.\\
&\left.\left.+\frac{625\log^2(z)}{48}+\frac{742433}{1944}\right)\right]\\
&+\mathcal{O}(\alpha_s^4)
\end{aligned}
$$

NNLL+NLO Results

- Resummed results at NNLL+NLO:

Gluon Jets (From Higgs)   Quark Jets (From e^+e^-)





- Distribution depends very sensitively on quark vs gluon!
- A fun example: For an adjoint gluino in $\mathcal{N}=1$, there is no LL

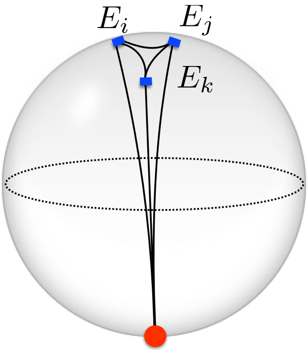
$$\frac{z}{\sigma_0}\frac{d\Sigma^{\mathcal{N}=1}_{NLL}}{dz} = \frac{3}{2}C_A\frac{\alpha_s}{4\pi} + \left(-4\zeta_3 + \frac{1417}{72}\right)C_A^2\left(\frac{\alpha_s}{4\pi}\right)^2 + \frac{(12\zeta_2 - 11)C_A^2\left(\frac{\alpha_s}{4\pi}\right)^2}{\left(1 + 3C_A\frac{\alpha_s}{4\pi}\ln z\right)}$$

- Sufficiently sensitive to collinear structure to have a qualitatively different LL structure for gluino (C_A) and gluons (C_A)!

EEC for Jet Substructure

- The EEC is a true collinear observable. It has a number of interesting properties for jet substructure:
  - It is single logarithmic.
  - Grooming does not modify the EEC.
    It is already groomed! $\implies$ Massive simplification for calculations.
  - It is a very sensitive probe of initiating parton (e.g. quark vs. gluon).
    Beyond C_A vs. C_F Casimir scaling.
  - It connects with some of the most well computed QCD quantities (twist-2 anom. dims.) $\implies$ extension to NNNLL in progress.

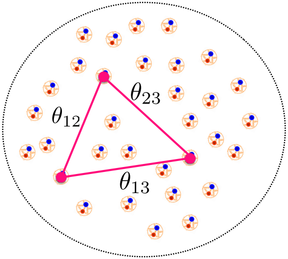
The Three Point Correlator



[Chen, Dixon, Luo, IM, Yang, Zhang, Zhu]

Three Point Correlators

- Jet Substructure calculations have primarily focused on two-particle type correlations (e.g. mass).
- Higher point correlators encode more interesting information about the internal structure of jets. See Jesse Thaler's Talk
- The three point correlator is a function of two cross ratios, r_1, r_2:



$$r_1 = \frac{\theta_{23}^2}{\theta_{12}^2}$$

$$r_2 = \frac{\theta_{13}^2}{\theta_{12}^2}$$

- Overall scaling with size of triangle determined by twist 2 spin 4 anomalous dimension.

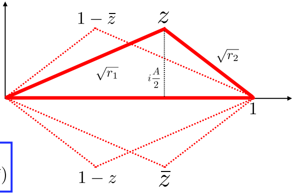
Triangles, Symmetries and Functions

- Parametrize unit triangle using a complex variable z:

$S_3 \times \mathbb{Z}_2$ Symmetry:

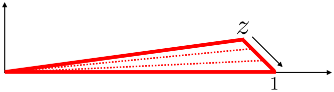
$$z \to 1-z \qquad z \to 1-\frac{1}{z}$$
$$z \to \frac{1}{z} \qquad z \to \frac{z}{1-z}$$
$$z \to \frac{1}{1-z} \qquad z \to \bar{z}$$

$$\boxed{\begin{aligned} r_1 &= z\bar{z} \\ r_2 &= (1-z)(1-\bar{z}) \end{aligned}}$$

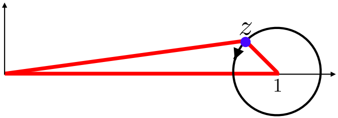


- Rigid analytic structure due to physical constraints:

OPE (Squeezed) Limit



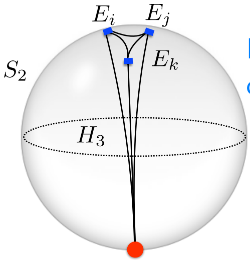
Vanishing Monodromies



- Lorentz group acts on celestial sphere as $\mathrm{SL}(2,\mathbb{C}) \implies$ result transforms as a conformal primary.

Jet Substructure and Hyperbolic Tetrahedra

- Result has an elegant interpretation: (Up to a few terms required to ensure behavior in limits) It is proportional to the volume in H_3 with points on the S_2 boundary (celestial sphere) at $(0, 1, z, \infty)$

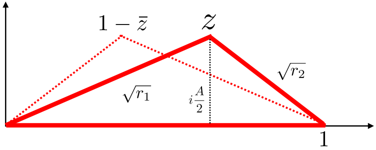


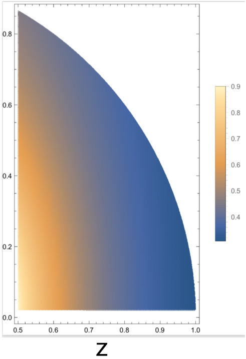
Lorentz Group acts on S_2 as $\mathrm{SL}(2, \mathbb{C})$ [Dirac]

- Scale dependence of volume governed by twist-2 anomalous dimensions $\Longrightarrow$ beautiful geometric picture of jet substructure!
- Remarkable (unexplored) hidden simplicity in the substructure of jets hints extension to higher points possible.

- Multi-particle correlations under analytic control!

Three Point Correlator Gluon Jet

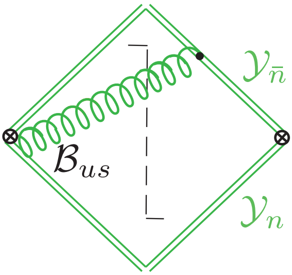




$$|1-z|^2 \frac{d\sigma}{dz}$$

- Would be fascinating to measure.
- Interesting as a probe of parton shower beyond $1 \to 2$ splittings.

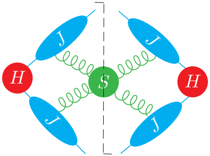
Extending Resummation Beyond Leading Power



[IM, Stewart, Vita, Zhu]
[See also: Beneke et al., Magnea, Laenen et al.]

Power Corrections for Event Shapes

- "Standard" factorization describes only leading term in $\tau \to 0$ limit.
- More generally, can consider expanding an observable in τ



$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n \sum_{m=0}^{2n-1} c_{nm}^{(0)} \left(\frac{\log^m \tau}{\tau}\right)_+ \quad \text{Leading Power (LP)}$$

$$+ \sum_{n=1}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n \sum_{m=0}^{2n-1} c_{nm}^{(2)} \log^m \tau \quad \text{Next to LP (NLP)}$$

$$+ \sum_{n=1}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n \sum_{m=0}^{2n-1} c_{nm}^{(4)} \, \tau \log^m \tau + \cdots$$

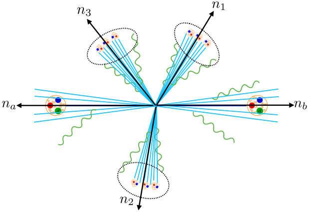
$$= \frac{d\sigma^{(0)}}{d\tau} + \frac{d\sigma^{(2)}}{d\tau} + \frac{d\sigma^{(4)}}{d\tau} + \cdots$$

- Open problem in QFT how to systematically describe corrections.
- Use Renormalization Group techniques to understand structure.

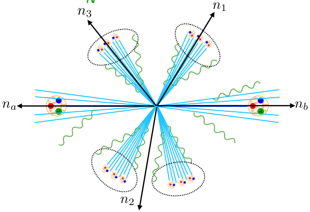
An Immediate Practical Application

- Jet observables can be used to resolve singularities for fixed order calculations of cross sections:

$$\sigma(X) = \int_0 d\mathcal{T}_N \frac{d\sigma(X)}{d\mathcal{T}_N} = \int_0^{\mathcal{T}_N^{\text{cut}}} d\mathcal{T}_N \frac{d\sigma(X)}{d\mathcal{T}_N} + \int_{\mathcal{T}_N^{\text{cut}}} d\mathcal{T}_N \frac{d\sigma(X)}{d\mathcal{T}_N}$$



Analytic NNLO calculation in singular limit



Numerical NLO calculation in resolved limit

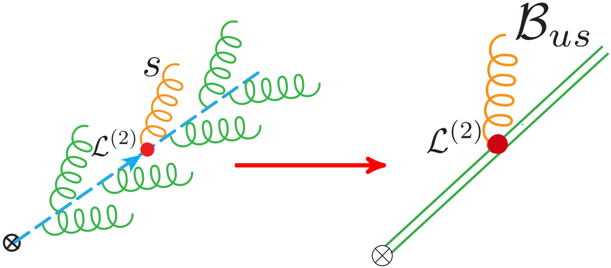
- If $\frac{d\sigma(X)}{d\mathcal{T}_N}$ can be computed as a power series, then this can enable efficient NNLO calculations for hadronic final states.

See talk by Markus Ebert

Gauge Invariant Factorization

[IM, Stewart, Vita, Zhu]

- Wilson lines are no longer sufficient at subleading powers.
- Can derive a complete basis of non-local gauge invariant operators which can decorate Wilson lines.



$$[Y_{n_i}^{(r)\,\dagger} iD_{us}^{(r)\,\mu} Y_{n_i}^{(r)}] \equiv T^a_{(r)} g \mathcal{B}^{a\mu}_{us(i)}, \qquad Y^\dagger_{n_i} q_{us} \equiv \psi_{us(i)}$$

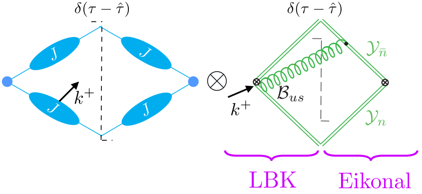
- Objects of interest are matrix elements Wilson lines decorated with non-local $\mathcal{B}^{a\mu}_{us(i)}$ and $\psi_{us(i)}$ fields.

Factorization at Subleading Power

[IM, Stewart, Vita, Zhu]

- This basis of operators allows factorization to be extended to any power in the soft and collinear expansion.
- Unlike at leading power functions are in general tied by a convolution along light cone directions:

$$\frac{d\sigma^{(2)}}{d\tau} = \int dk^+$$



$$\mathrm{tr}\langle 0|\mathcal{Y}_{\bar{n}}^T(x)\mathcal{Y}_n(x)\bar{n}\cdot\mathcal{B}_{(n)}(x)\delta(\tau-\hat{\tau})\mathcal{Y}_n^T(0)\mathcal{Y}_{\bar{n}}(0)|0\rangle = \int \frac{d^4 r}{(2\pi)^4} e^{-ir\cdot x} S_{LBK}(\tau, r)$$

- At tree level reproduces standard Low-Burnett-Kroll (LBK) theorem.
- Power suppressed logarithms associated with the LBK operator can be resummed by renormalization group evolution.

Renormalization of LBK Operator

[IM, Stewart, Vita, Zhu]

- Renormalization of the LBK operator involves mixing into a new Wilson line operator:

$$\bar{n}\cdot\mathcal{B}_{(n)}(x)\delta(\tau-\hat{\tau})$$

$$\downarrow$$

$$S_\theta(\tau,\mu)=\mathrm{tr}\langle 0|\mathcal{Y}_{\bar{n}}^T(0)\mathcal{Y}_n(0)\theta(\tau-\hat{\tau})\mathcal{Y}_n^T(0)\mathcal{Y}_{\bar{n}}(0)|0\rangle$$

- Leads to a 2×2 evolution equation.

$$\mu\frac{d}{d\mu}\begin{pmatrix} S_{LBK} \\ S_\theta \end{pmatrix}=\begin{pmatrix} \gamma_{LBK\to LBK} & \gamma_{LBK\to\theta} \\ 0 & \gamma_{\theta\to\theta}\end{pmatrix}\begin{pmatrix} S_{LBK} \\ S_\theta \end{pmatrix}$$

- Remarkably, the leading logarithms associated with the LBK operator exponentiate!

$$S_{LBK}(\tau,\mu)=\theta(\tau)\gamma_{LBK\to\theta}\log\left(\frac{\mu}{\tau}\right)e^{\frac{1}{2}\frac{\alpha_s}{4\pi}\Gamma^g_{\mathrm{cusp}}\log^2\left(\frac{\mu}{\tau}\right)}$$

Subleading Power Resummation for Beam Thrust

- Allows for the resummation of power suppressed logarithms for hadron collider event shapes.
- e.g. Resummation for power suppressed contribrutions to beam thrust, τ_0 in $gg \to H$: [IM, Schunk, Stewart, Tackmann, Vita, Zhu]

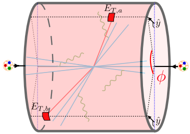
$$\frac{\mathrm{d}\sigma^{(2)}_{\mathsf{LL}}}{\mathrm{d}Q^2\mathrm{d}Y\mathrm{d}\tau_0} = \hat{\sigma}^{\mathrm{LO}}(Q)\left(\frac{\alpha_s}{4\pi}\right)4C_A\theta(\tau_0)\log(\tau_0)e^{-\frac{\alpha_s}{4\pi}4C_A\log^2(\tau_0)} \cdot \left[2f_g(x_1)f_g(x_2) - x_1 f_g'(x_1)f_g(x_2) - f_g(x_1)x_2 f_g'(x_2)\right]$$

- Beyond Leading Power, derivatives of the PDFs enter.
- Immediately predicts power suppressed terms for subtractions.

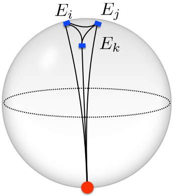
- Greatly extends scope of resummation based techniques beyond leading power, but much work to be done on general understanding of resummation at subleading powers.

Summary

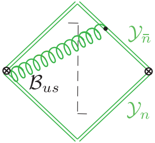
- Simplicity of TEEC Allows Hadron Collider Dijet Event Shapes at NNNLL



- Jet Substructure Exhibits Remarkable Unexplored Simplicity



- Simplicity in Singular Limits Persists Beyond Leading Power



Thanks!